Robust shape optimization of NURBS based acoustic reflectors using stochastic search techniques

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ABSTRACT

This paper demonstrates the successful application of stochastic search techniques to the shape optimization of acoustic reflectors. An optimizer plugin - Stochastic Optimizer for Acoustic Reflectors (SOAR) - was developed. In order to carry out the ray tracing component required for optimization, the NURBS Ray Analysis Tool (NRAT), developed by the author’s colleagues was utilized. Both these plugins worked in conjunction with Rhinoceros to design three dimensional NURBS surfaces to optimize Reflector Efficiency \( \rho \) and Audience Distribution \( \Delta \). It was found that optimizing the latter objective function provides uniform distribution of sound rays on the audience plane. The robustness of the reflector was improved by accounting for noise in the source location using robust optimization techniques. This technique could also be extended to account for manufacturing tolerances by applying perturbations to independent control points about the original location. Surrogate modelling was also implemented but was found to produce suboptimal results due to the highly multimodal behaviour of the objective function space.

1 INTRODUCTION

Throughout history, acoustical engineers have used many techniques to aid in the design of auditoriums. These include wave tables, spark photography, scale models with scaled frequency testing, and optical model testing.\(^1,2\) Since then, mathematical computer modelling using ray-tracing, beam-tracing, source image method and radiant exchange methods have become practical.\(^3\) However, currently available acoustic softwares do not allow for efficient manipulation of curved surfaces with the sophistication for accurate mathematical depiction as well as the simplification for usability. Representing surfaces using the non-uniform rational basis spline (NURBS) model allows for precise mathematical definition of surfaces while also providing the user with control points that are easy to manipulate.\(^4\)

NURBS can be quite useful in the design of acoustic reflectors. These surfaces serve to mirror sound originating from one part of the space to the other in order to improve and optimize acoustic parameters.\(^5\) Flat plate reflectors can be limited to where they can potentially direct the sound rays. There have also been some concerns of flat plate reflectors introducing image shift requiring the need for diffusion.\(^6\) In most engineering applications, the design space is heavily constrained by lighting elements or architectural design elements. Curved reflectors can potentially overcome this limitation and direct rays to hard to reach sections of the audience plane.
Without acoustic reflectors, sound rays may bounce around aimlessly. In doing so, they may reach the audience after the 80ms time frame or become attenuated. This decreases clarity (C-80)\textsuperscript{5} and deteriorates the quality of music as perceived by listeners. Therefore, the requirement for reflected sound to reach the audience within 80ms of the direct sound for music puts a fundamental constraint on how far these reflectors can be located. Every ray traced during the optimization process has a time of travel associated with it. Rays that reach the audience plane within the first 80ms have been used in this study. In other words, optimization takes place using only useful sound energy.

There have been numerous applications where reflectors were used to improve the sound in auditoria. Jeon \textit{et al} used reflectors to enhance the sound strength (G) and spatial responsiveness of symphonic music in the Grand Theatre of the Sejong Performance Arts Center.\textsuperscript{7} In 2007, a renovation of the Queen Elizabeth Theatre in Vancouver saw the use of lateral reflectors in order to improve the spatial sound of the theatre.\textsuperscript{6} An implementation of this design showed that many acoustic parameters were improved.\textsuperscript{8} Guangzhou Opera House uses curved side walls to generate useful early reflections.\textsuperscript{9}

In designing a curved acoustic reflector, engineers have to manually tweak the control points of a NURBS surface in order to achieve the desired coverage on the audience plane. This can be a time-consuming process. This research develops a method to optimize the distribution of sound rays on the audience plane by manipulating the shape of a given reflector in an auditorium. The user will provide spatial constraints for the reflector along with the audience area and the source location. The present research will focus on developing and applying optimization algorithms to determine a reflector shape that will optimize performance metrics. Optimization techniques, although with much fewer applications, have been applied in auditorium design. Combinatorial simulated annealing and gradient-based techniques were applied to optimize an objective function derived from Beranek’s objective acoustic parameters.\textsuperscript{10} Genetic algorithms were applied to reflector design\textsuperscript{7} along with a publication that focuses on optimizing the shape of reflectors in a fan-shaped hall.\textsuperscript{11} However, to the knowledge of the author, most of these implementations are done in softwares that are not widely used in the design of auditoriums. In addition, the applications of optimization techniques are limited to specific planar geometry\textsuperscript{7} or discrete set of allowable planar geometry and transformations.\textsuperscript{10}

This research aims to remove the restriction of specific geometry as well as the need to optimize over a set of discrete geometry and transformations. Furthermore, the research attempts to optimize curved surfaces removing the need for manual manipulations of these complicated structures. In this way, unique curved geometry can be created in spaces without manually handling these complicated surfaces.

The potential use of this will be to assist in designing reflectors for new or existing auditoriums around the world without the need to have engineers invest a lot of time. Reflectors can be used to improve the sound quality of a space without changing the entire space, a costly process. Reflector design using curved surfaces is becoming popular among leading companies and the outcome of this research can be used in the acoustic industry. Due to the stochastic nature of the optimization algorithm, new and more interesting reflector shapes will give auditoriums a unique signature. Furthermore, the result of this study can be used in other applications that utilize NURBS to represent surfaces.
2 PROBLEM AND METHODOLOGY

2.1 Problem

The process of designing a space that acts as the ideal sound propagator has been an active area of research. The fundamental research question to be tackled in this study is the following: *Given a source of sound, a receiver area, and a reflector surface, what is a suitable shape for the reflector in a constrained design space that seeks to optimize the distribution of sound rays - originating from a point source - on an audience plane.*

![Figure 1: An auditorium showing a source, reflector, and a receiver. The problem deals with manipulating the shape of the acoustic reflector to achieve an optimal distribution of sound rays on the audience plane.](image)

2.2 Tools and Techniques

In order to predict where the sound rays will be directed to in the audience plane, the NRAT plugin developed by Aerocoustics Engineering Limited was used. This plugin works in Rhinoceros and is able to trace sound rays originating from a spherical source location as it interacts with other surfaces in the model space. As part of this research, a plugin known as *Stochastic Optimizer for Acoustic Reflectors* (SOAR) was developed. SOAR works in conjunction with NRAT. NURBS based parameterization of the acoustic reflector shapes was utilized to remove many of the geometrical restrictions placed on the possible shapes of reflecting surfaces in the past. The optimization is no longer a discrete one but by freely manipulating the control points, a continuous range of geometries in the design space can be explored. In this way, the design is no longer restricted to a discrete number of possibilities. In addition, a stochastic approach outlined in Section 2.4 has been taken to remove any bias in the search process. This approach has also helped to traverse the highly multimodal objective function space as seen in Figure 4.
2.3 Objectives Function Formulation

**Reflector Efficiency:** The first objective function considered in this work is the *Reflector Efficiency*, $\rho$ which can be defined as follows:

$$\rho = 1 - \frac{R_{\text{success}}}{R_{\text{total}}}$$  \hspace{1cm} (1)

The total number of rays, $R_{\text{total}}$, is the number of intersecting iso-curves on the NURBS surface. The number of rays that are considered successful, $R_{\text{success}}$, are those that manage to reach the audience plane from the reflector. Therefore, for a given reflector, when $\rho$ equals 0, every single part of the reflector is useful in sending rays to the audience plane.

**Audience Distribution:** To compute this objective function, a new algorithm was developed to determine the exact distribution of sound rays on the audience plane. The audience plane was compartmentalized into sections. The number of sound rays falling on each section of the audience plane was then used to develop a distribution equation of the sound rays on the audience plane. The average number of rays falling on the surface was calculated. This mean value was then compared to the number of rays falling on each seat to determine the objective function as follows,

$$\sum_{i=1}^{N_{\text{seats}}} (R_i - \bar{R})^2$$  \hspace{1cm} (2)

In this formulation, the objective function can be driven to a minimum if $R_i = \bar{R}$ for all $i$. In this way, the mean value, $\bar{R}$, will be equal to the number of rays in each section $R_i$ cancelling out all contributions to 0. In other words, the distribution of sound rays is a constant function over the audience plane ensuring a uniform distribution. However, this can be easily achieved if there are no rays hitting the audience plane in which case, $\bar{R} = R_i = 0$. Therefore, certain constraints have to be placed in order to ensure this does not happen. But, instead of placing constraints, the objective function was revisited and modified to inherently take this into account. The modified objective function for the *Audience Distribution* criteria can be expressed in the following way,

$$\Delta = \sum_{i=1}^{N_{\text{seats}}} \left( \frac{R_i - \bar{R}}{\bar{R}} \right)^2$$  \hspace{1cm} (3)

In this formulation, if the average number of rays on the audience plane decreases, the objective function increases. Therefore, the optimization process is not only driven to uniformity in the audience plane but it is also driven to maximize the number of rays being utilized. Whereas in (2), the objective function would benefit from having a low mean.

2.4 Optimization

A popular gradient-free stochastic optimization technique, simulated annealing, provides a means of finding a global minimum in a highly multimodal objective function space. Using this algorithm, a global minimum can be guaranteed in the probabilistic sense. Given sufficient amount of time, the algorithm will eventually find the global minimum without getting stuck in a local minima. Due to the hill climbing nature of the algorithm, in order to reach convergence, computation time has to be increased. The simulated annealing algorithm was applied to the problem of optimizing the shape of acoustic reflectors in order to attain an optimal redirection of sound rays from a sound source to
the audience. Nothing is known about the features of the objective function and so this stochastic search technique provides an unbiased approach.\textsuperscript{14} Due to the relatively fast computational time required for ray tracing, objective functions can be evaluated without being bogged down by the need to solving expensive wave formulations.

2.5 Simulated Annealing in Practice

In order to apply simulated annealing algorithm to practical problems, a number of parameters must be introduced. These parameters govern the stochastic algorithm and is important in finding good solutions within a given amount of time.

**Start Temperature:** As mentioned in Algorithm 1, the hill climbing nature of simulated annealing is governed by a temperature parameter, $T$. The start temperature, $T_0$, depends on the objective function of the problem as well as, on average, what kind of moves are being performed. In what follows, an acceptable start temperature for any problem can be calculated using,

$$
T_0 = \frac{\Delta f^+ \left( \ln \frac{m_2}{m_2 \chi_0 + (1 - \chi_0) m_1} \right)^{-1}}
$$

where $m_1$ is the number of accepted moves ($\Delta f < 0$), $m_2$ is the number of rejected moves ($\Delta f > 0$), $\chi_0$ is the initial acceptance criteria, and $\Delta f^+$ is the mean value of all $\Delta f > 0$. The $\Delta$ notation used here should not be confused with that used to define the Audience Distribution objective function in (3). This formulation is valid for any objective function $f$.

Of course, if $T$ is fixed at this $T_0$, the optimization process does not converge. Therefore, a method of reducing the temperature is required.

**Cooling Schedule:** There are a number of ways to design the cooling schedule.\textsuperscript{13} In this study, a simple method of exponentially cooling the temperature was used. At every iteration, the temperature was reduced by $\tau$ using $T_{k+1} = \tau T_k$.

**Move Generation:** The perturbation mechanism can be generated in many ways. A random unit perturbation vector, $p_k$, can be generated consisting of the same number of elements as the design variables. A step size, $\epsilon_k$, can then be used to produce, $\epsilon_k p_k$. This can then be used to perturb the design vector, $x_k$. The only thing to note here is that the perturbation vector shall not move the design vector out of the design space, $\Omega$. In order to explore the space thoroughly, the step size needs to be large. However, if the step size remains large, it becomes insensitive to local gradients. For this reason, the step size was adaptively reduced as the search progressed. It has been shown in many cases that the cooling schedule used for the temperature should also be used for move generation.\textsuperscript{15} However, due to the nature of the problem, the temperature is orders of magnitude higher than the control volume dimensions. Therefore a factor, $\beta$, was used to control how frequently the move step size is reduced. This is given by $\epsilon_{k+\beta} = \tau \epsilon_k$.
2.6 Developing an Algorithm

A simple representation of the implemented simulated annealing algorithm is shown in Algorithm 1.

Initiate an iteration counter, \( k = 0 \);
Determine the objective function, \( f(x_0) \), of an initial state.;
Determine an appropriate start temperature, \( T_0 \);.
Set a cooling parameter between 0 and 1, \( \tau \);
while Convergence is not achieved do
    Determine a random perturbation vector, \( p_k \);
    Choose a step size, \( \epsilon_{k+\beta} = \tau \epsilon_k \);
    Determine the objective function, \( f(x_k + \epsilon_p_k) \);
    if \( f(x_k + \epsilon_p_k) < f(x_k) \) then
        The perturbed state becomes the current state, \( x_{k+1} = x_k + \epsilon_p_k \);
        Save the best perturbation, \( x_{best} = x_{k+1} \)
    else
        Calculate a probability based on the Metropolis criterion,\(^{16}\) \( P = e^{\frac{f(x_k) - f(x_k + \epsilon_p_k)}{T}} \);
        if \( P \) is greater than a randomly generated number, \( R \) between 0 and 1 then
            The perturbed state becomes the current state, \( x_{k+1} = x_k + \epsilon_p_k \);
        else
            The perturbation is rejected and the next state remains the same, \( x_{k+1} = x_k \);
        end
    end
Reduce the temperature using an annealing schedule of the form \( T_{k+1} = \tau T_k \);
Increase the iteration counter, \( k = k + 1 \);
end
Return \( x_{best} \)

Algorithm 1: General simulated annealing algorithm.

2.7 Robust Optimization

A common problem in optimization is that in the search for finding better objective functions, robustness may be compromised\(^{17}\) - the solution may be too sensitive to product tolerances or other disturbances such as the source not being fixed.

Source Perturbation: Rarely is the source fixed unless the source is an electronic sound source such as a speaker. In live performances, sound sources inevitably move from one location in the stage to the other. If a reflector was too sensitive to these changes, it may become suboptimal in directing sound waves to the audience. Inevitably, accounting for these disturbances become a priority. In optimizing the reflector, the source was at first kept at a user defined location. However, the algorithm was altered to take the noise in the source location into account. For every perturbed reflector, the source location was stochastically disturbed in a circular neighbourhood \( \xi \), around the actual source location. The radius, as well as the number of source trials, \( S_N \) can be specified by the user. In this way a vector of objective function was generated. Revisiting the objective function formulation presented in (3), a further alteration was made. The values calculated at each source location can be denoted by \( \Delta_j \forall j = 1 ... S_N \). The final objective function for a given reflector shape can be computed in various ways.\(^{17}\)

In this study two methods to improve the robustness in the objective function was utilized. The
first one is a simple arithmetic average while the second one is a maximum value. The latter case is more robust since the worst objective function is minimized which by definition minimizes any other perturbed source location in the circular neighbourhood $\xi$. For example, assume Reflector A gives a $\Delta_0 = 80$ while Reflector B gives $\Delta_0 = 75$. However, when the source location is disturbed $S_N$ times, Reflector A gives a worst case $\Delta_{max} = \Theta = 90$ while Reflector B gives a $\Delta_{max} = \Theta = 100$. In non-robust optimization, the resulting solution would be Reflector B while in this modified approach the resulting solution would be Reflector A which is a more robust reflector, although it under-performs at the original source location. The first method of robustness can be calculated using,

$$\Theta = \sum_{j=1}^{S_N} \frac{\Delta_j}{S_N}$$

In circumstances requiring increased level of robustness, the following objective function can be used:

$$\Theta = \arg \max_{j \in \xi} \Delta_j$$

The final minimization problem that was solved in this study can be stated as,

$$x^* = \arg \min_x \Theta(x)$$

where $x^*$ is the control points representing the optimal reflector and $\Theta(x)$ is the robust objective function that is minimized.

Manufacturing Tolerances: A reflector can not be built perfectly as designed. During manufacturing or construction, the reflector may be subject to tolerances. A higher tolerance is usually put on these processes in order to reduce the cost significantly. A technique similar to the one utilized to account for source perturbations can be developed to account for manufacturing tolerances. By perturbing the control points of a reflector in a sphere around the original location, a set of objective functions can be generated. This can then be used to specify or account for manufacturing tolerance such that the reflector’s performance is not negatively affected during construction.

3 RESULTS

3.1 Comparison of the Objective Functions

The optimization algorithm was run on the three objective functions described in Section 2.3. The design volume, source location, and the initial condition was set to be equal in the following studies. When only the Reflector Efficiency was used, the distribution on the audience plane was seen to cluster. This is illustrated on the right in Figure 2.

The objective function was then modified to take the audience distribution into account. Runs were conducted on the modified objective functions formulated in (2) and (3). The results are shown in Figure 3.

From Figures 2 and 3, the importance of the objective function can be readily seen. When the objective function was $\rho$, the algorithm clustered the sound rays on the audience surface leading to highly uneven distributions. When the objective function was changed, due to the poor structure of the function, there were no significant improvements. But, clustering was minimized (left of
Figure 2). A modified version of the objective, \( \sigma \), proved to not only increase uniformity but also improve the number of successful rays on the audience plane.

Figure 2: Optimization results using the first objective function. Result from the same run after 50 iterations is shown on the left and after 500 iterations is shown on the right. It can be seen that the earlier result is more uniform while later on the results are less uniform and more dense. The algorithm chooses the clustered distribution as it provides a higher Reflector Efficiency \( \rho \) which is the objective function in this run. The solution on the right was attained in approximately 8 minutes.

Figure 3: Optimization results using the second objective function (left) and the improved objective function \( \Delta \) (right) after 14000 iterations. It can be seen that the distribution, although even, is sparse for the second objective function while the improved objective function yields a result that is both uniform and dense.

3.2 Study of the Objective Function

The objective function formulated in (3) was studied in detail to determine its characteristics. Using data obtained from the run shown in Figure 5, three dimensional cutaways of the objective function space was plotted against two design variables for visualization purposes. Figure 4 helps to visualize the highly multimodal characteristics of the objective function. This plot justifies the use of simulated annealing which has helped to traverse this multimodal function in order to arrive at better function values. If gradient based techniques were used instead, depending on the initial condition, the optimization algorithm may have lead to a local minima producing suboptimal reflectors.
Figure 4: This plot shows the objective function as a function of two control points on the NURBS surface. The entire surface consisted of 27 control points that were free to move in three dimensions. What is shown here is just the z direction perturbation. The run consisted of 20000 iterations. This plot was developed using a natural interpolation scheme from the sampled data points. Notice that the objective function has been flipped in order to show the characteristics clearly. Shown in pink is the minimum found in the optimization process.

3.3 Influence of the Cooling Parameter

A reflector with 27 control points was optimized with various cooling parameters, $\tau$, introduced in 2.5. Three values of $\tau$ are presented: 0.99, 0.999, and 0.9999.

Figure 5: Best reflector (left) and convergence plot (right) with $\tau = 0.99$
From Figures 5-7, as the system is cooled slower, the probability of finding a global minimum increases. In the case of $\tau = 0.99$ the process prematurely gets stuck in a local minimum. In the opposite case of $\tau = 0.9999$, a better solution is found. The distribution is relatively even as well as dense as required. However, the number of iterations increases significantly. A careful balance is required for the cooling parameter such that good solutions are found quickly.

### 3.4 Effect of Control Points

Increasing the number of control points allow for more interesting and flexible reflector design. However, it was found that when the number of control points increased, the number of iterations required for finding good distributions also increased. This is shown in Figures 5 to 7. In these runs a 3x9 grid of control points were used as the design variables. The problem dimension is 81 since each control point can be moved in three dimensions. Due to the high dimensional attribute of this problem, many modes were present in the objective function space as illustrated in the simple two dimensional cutaway of the objective function shown in Figure 4. In order to traverse this space, simulated annealing required a very slow cooling process. If it was cooled quickly as shown in Figure 6, the reflector converged to a local minimum producing poor uniformity and density in the audience plane. When the optimization process was given more freedom to explore the objective function better results were obtained. This required $\tau$ to be increased causing the temperature decay to be slower. This means the optimization process is more willing to climb uphill helping it to overcome the many number of modes in the objective function space. However, it appears...
that according to Figure 7, the cooling parameter needs to be increased even more in order to find better solutions so that the distribution is more uniform.

In the next study, the number of control points were reduced to a 3x3 grid of control points giving a dimension of 27. This may be detrimental in cases requiring a more complex reflector. However, in the simple problem shown in Figure 8 it was found that a lower number of control points not only provides the necessary uniformity in the audience plane but was also able to do this in 600 iterations compared to 350000 iterations as required by a 3x9 grid NURBS reflector. The number of control points should be carefully chosen to account for problem dynamics while trying to minimize the required computational resources.

3.5 Robustness against Source Perturbations

![Objective Function vs. Iterations](image1)

![Objective Function Distribution](image2)

**Figure 8:** Convergence plot of the robust objective function. Seen on the left is the worst case distribution. The black line on the graph represents the robust objective function used during minimization. The dashed blue line shows the best objective function values found during the source perturbation. The bar graph shows the distribution in objective function for the final solution. The turquoise circle represents the neighbourhood of source perturbation.

Robust design can be enabled by the user for the whole optimization process or alternatively at a later stage. The former case is recommended although expensive. The latter option can be beneficial if computational costs are limited. The total computational time for the simulation shown in Figure 8 below was approximately 98 minutes when robust design was enabled with the number of source trials, $S_N = 10$. If robust optimization was disabled, the same run would have taken approximately 15 minutes. The following result shows that the objective function $\Theta$ is not hugely...
sensitive to source perturbation in the neighbourhood $\xi$ with radius equal to 1m. The distribution shown in Figure 8 is the worst case scenario for this particular problem since $\Delta_{\text{max}}$ was minimized. Probabilistically, source perturbations will result in distributions that are better than what is shown in Figure 8. Extending this further, the whole stage can be be used as the source perturbation neighbourhood, $\xi$. Although, this maybe a very expensive process, in many applications it will be extremely useful.

4 CONCLUSION

The research successfully developed a tool kit, SOAR, that performs stochastic search optimization on a given NURBS surface. SOAR, although developed to work with Rhinoceros, could be easily ported to other softwares that utilize NURBS. Various reflectors were designed and optimized to yield excellent uniform audience distribution without manual intervention. It was found that the choice of using simulated annealing was extremely fruitful in traversing the highly modal objective function space. In addition, using Audience Distribution $\Delta$ as the objective function yielded the required uniform results. In order to safeguard against any disturbances in the source position, SOAR also implements robust shape optimization. With this mode enabled, the optimization algorithm minimizes either the average objective function or the maximum objective function found in a neighbourhood around the original source location. This technique could also be extended to account for manufacturing tolerances by perturbing the control points of the NURBS surface around a sphere centred on the original location of the control point. Next, it was found that having many control points required extensive amount of iterations while providing new and interesting reflector shapes. On the other hand, having a fewer number of control points reduces the number of iterations required to find good solutions while suffering from a lack of innovative and interesting solutions. However, a large number of control points were required in some problems in order to give the reflector enough flexibility to direct the sound to hard to reach areas of the audience plane. The majority of the simulations was conducted on a laptop. By using a more capable computer, SOAR can optimize reflectors with a higher number of control points as well as sample a larger number of source disturbance locations. An extreme application of SOAR would be to optimize a reflector spanning the whole ceiling while ensuring that the reflector is robust enough to handle sources at any location on the entire stage. In conclusion, the research conducted in the development and study of SOAR demonstrates that it can be used by acousticians and engineers seeking to optimize acoustic reflector designs in auditoria.

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