ABSTRACT

As an alternative for the common geometric room acoustical models, a wave theory-based model has been developed derived from the so-called WRW scheme, often used in seismic modelling\(^1,2\). In this model, the wave properties of sound are appropriately taken into account. The proposed model is similar to the Boundary Element Method (BEM). Other than in BEM, in the WRW scheme non-locally reacting room boundaries can be included in the modelling process using only the fluid domain. The results of sound field modelling in rooms with locally as well as non-locally reacting boundaries are presented. The differences are analysed and their relevance is discussed.

1 INTRODUCTION

The WRW scheme was developed for wave field modelling in the context of seismic oil and gas exploration, i.e., related to the layered structure of the earth bottom. The scheme is based on the Huygens principle, which is quantitatively described by the Kirchhoff-Helmholtz integral and specified for plane boundary configurations by the Rayleigh representation integrals. The derivation of the corresponding mathematics is given in a paper presented at the 21st ICA\(^3\). The development of this WRW scheme for room acoustics is to create a tool that allows non-locally reacting (angle-dependent) boundary conditions to be simulated in a one-domain wave based simulation. For BEM or FEM simulations it is necessary to couple the acoustic domain to a structural domain, which increases calculation time. For the WRW scheme there is no difference in calculation time using locally or non-locally reacting boundary conditions.

2 THE WRW SCHEME FOR A RECTANGULAR ROOM

In the ICA paper\(^3\), it is shown how the WRW scheme can be adapted for modelling the wave field, in terms of sound pressure, in a rectangular room enclosed by six plane boundaries \((b_1\) to \(b_6)\). A 2D cross section of such room is shown in Figure 1; \(s_x\) and \(d_x\) denote sources and detectors respectively. A summary, and a clarification of the notation, is given in the following.

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The WRW scheme can elegantly be represented in vector-matrix notation, where all variables are given in the space-frequency domain. The resulting sound pressure at the detector positions is given by vector $\vec{P}$, and is the sum of a direct component and a reflective component, as shown in Figure 2.

The contribution to the wave field by the reflective part can be defined as an iterative process, which starts with propagation from the sources to the walls followed by a reflection and propagation to the other walls and finally reaches the detector points. The calculation of the reflected wave field is schematically shown in the top part of Figure 2. The matrix operators that include all the reflection and propagation properties for the complete geometry are defined as follows. The source vector $\vec{S}$ includes the number of sources and their strength for the specific frequency $\omega$ and is defined as (with the total number of sources $i$):

$$\vec{S} = \begin{bmatrix} S_1(\omega) & S_2(\omega) & \cdots & S_i(\omega) \end{bmatrix}^T$$  \hspace{1cm} (1)

The matrix operator for the propagation from sources to detectors $W_{ds}$, sources to the boundaries $W_{bs}$ and boundaries to the detectors $W_{db}$ are defined as (with the total number of sources, detectors and/or boundaries indicated as $i$ and $j$):

$$W_{xy} = \begin{pmatrix} W_{s_1x_1} & W_{s_1x_2} & \cdots & W_{s_1x_i} \\ W_{s_2x_1} & W_{s_2x_2} & \cdots & W_{s_2x_i} \\ \vdots & \vdots & \ddots & \vdots \\ W_{s_jx_1} & W_{s_jx_2} & \cdots & W_{s_jx_i} \end{pmatrix}$$  \hspace{1cm} (2)
The boundary properties are collected in one matrix $R$, where on the diagonal the matrices of separate boundary properties $R_b$, are placed:

$$
R = \begin{pmatrix}
    R_b & 0 & \cdots & 0 \\
    0 & R_b & & \\
    \vdots & \ddots & \ddots & \\
    0 & \cdots & 0 & R_b
\end{pmatrix}
$$

(3)

Propagation between boundaries is collected in the matrix $W$, which includes the individual propagations between the boundary parts (with $N$ the total number):

$$
W = \begin{pmatrix}
    W_{b_1b_1} & W_{b_2b_1} & \cdots & W_{b_Nb_1} \\
    W_{b_1b_2} & W_{b_2b_2} & \cdots & \vdots \\
    \vdots & \ddots & \ddots & \vdots \\
    W_{b_1b_N} & \cdots & W_{b_Nb_N}
\end{pmatrix}
$$

(4)

The matrix components $W_{pp}$ represent the interaction of the boundary with itself. This can be interpreted as propagating bending waves. In general cases these matrices will be zero $W_{pp} = 0$. Furthermore, it can be easily seen that $W_{pq} = W_{qp}$.

As shown in the ICA paper, the reflected wave field at the detector locations in case of one reflection with multiple boundaries is calculated by:

$$
\bar{P}_1 = [W_{db}RW_{bs}]\bar{S}
$$

(5)

and if this is extended to the $m^{th}$-order reflection, the expression in equation (5) can be generalized to:

$$
\bar{P}_m = [W_{db}(RW)^mRW_{bs}]\bar{S}
$$

(6)

The resulting reflected wave field at the detector locations $\bar{P}_{refl}$ is given by the summation of all reflection orders $M$, which gives the following expression in case $M = \infty$:

$$
\bar{P}_{refl} = \sum_{m=0}^{\infty} [W_{db}(RW)^mRW_{bs}]\bar{S} = [W_{db}(I - RW)^{-1}RW_{bs}]\bar{S}
$$

(7)

where use is made of the fact that the summation is a Neumann series, which can be written as a matrix inversion. The matrix $I$ here is the unity matrix. Furthermore, it is necessary to add the direct wave field $\bar{P}_{dir}$ from the source(s) to the detector(s) to get the total wave field $\bar{P}$ at the detector(s), schematically shown in the bottom part of Figure 2. The total wave field is therefore given by:
\[
\tilde{P} = \tilde{P}_{\text{dir}} + \tilde{P}_{\text{refl}} = \left[ W_{ds} + W_{dh} \left( I - RW \right)^{-1} RW_{bs} \right] \tilde{S}
\]

3 **LOCALLY AND NON-LOCALLY REACTING BOUNDARIES**

A reflector is called locally reacting when pressure excitation at a certain point only causes a reaction (i.e., reflection) in that same point. This means that the reflectivity function in the space domain is a delta pulse, as illustrated in the left part of Figure 3.

![Figure 3](image-url)

**Figure 3**: Reflection function in space-frequency \(R(x, \omega)\) and angle-frequency \(R(k_x, \omega)\) domain for locally reacting (left) and non-locally reacting (right).

This means that, in equation (3), for a locally reacting boundary the submatrix \(R_b\) will be a diagonal matrix.

Usually, the reflection coefficient of a boundary is given as a function of incidence angle \(\alpha\). For a certain (angular) frequency \(\omega\), this angle unambiguously corresponds with the wave vector component (or wave number) along the boundary surface:

\[
k_x = \frac{\omega}{c} \sin \alpha
\]

The reflectivity function in the space-frequency domain \(R(x, \omega)\) and the reflection coefficient in the wave number-frequency domain \(R(k_x, \omega)\) are related by a spatial Fourier transform:

\[
R(k_x, \omega) = \int_{-\infty}^{\infty} R(x, \omega) e^{-i(k_x x)} dx
\]

such that for a locally reacting reflector the reflection coefficient in the wave number- (or: angle-) frequency domain is constant, i.e., angle-independent. This is illustrated in the right part of Figure 3.

Usually, the reflection coefficient of a practical room boundary is angle-dependent. This means that in that case the spatial Fourier transform of \(R(k_x, \omega)\) will result in a spatially extended reflectivity function \(R(x, \omega)\): excitation of a boundary point will cause a reaction also in surrounding points and the reflectivity function can be interpreted as a spatial convolution operator. This is illustrated in Figure 3, where a block function was chosen for \(R(k_x, \omega)\).
In eq. (3), for a non-locally reacting boundary the submatrix $R_b$ will be a band matrix as given by eq. (11):

$$
R_b = \begin{pmatrix}
R(x_{11}, \omega) & R(x_{12}, \omega) & \cdots & 0 \\
R(x_{21}, \omega) & R(x_{22}, \omega) \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & R(x_{NN}, \omega)
\end{pmatrix}
$$

(11)

4 SIMULATION RESULTS

Three simple 2D configurations are simulated with both locally and non-locally reacting boundary properties. For the locally reacting case a fully reflective boundary is chosen and for the non-locally reacting case the block-function of the right plot in Figure 3 is applied. The results are calculated in the frequency domain, as previously explained, but shown in the time domain (by applying an inverse Fourier transform).

4.1 Single wall configuration

The first simulation is for a single wall with in front two possible source positions as shown in Figure 4.

Figure 4: Single wall configuration with sources (x) and detectors (•).

In the results from the single wall in Figure 5 the difference between source position 1 and 2 is clearly visible. For position 1 the direct wave field hits the wall perpendicularly and for position 2 under an oblique angle. It can be clearly seen that the first reflection for source position 2 shows more amplitude between a locally and non-locally reacting boundary condition than for source position 1. This is also in concurrence with the given angle-dependent function of the reflection coefficient.
4.2 L-shape wall configuration

The second simulation is performed with an L-shaped wall, where it is possible to show the influence on a second order reflection from the sidewall via the back wall (Figure 6).

The results of this simulation show the edge diffraction (arrow a in Figure 7), the first order reflection of the back wall (arrow b in Figure 7) and the second order reflection from the sidewall via the back wall (arrow c in Figure 7). Examining the difference it can be concluded that the second order reflection in the locally reacting case is significantly stronger than that of the non-locally reacting one. This means that simulating the sound field neglecting the non-locally reacting properties could predict coloration effects at non-central listening positions that do not occur in reality.
Figure 7: Microphone array responses for the locally and non-locally reacting boundary conditions and their difference. It shows the edge diffraction (a), first reflection of the wall parallel to the microphone array (b) and the second order reflection of the sidewall (c).

4.3 Closed space (room)

As a third case a closed space is simulated which contains one source and a microphone array as shown in Figure 8.

Figure 8: Closed space (room) configuration with source (x) and detectors (·).

What can be concluded from this simulation is that the influence of the non-locally reacting boundary condition is only important for the first two reflections. This can be observed in the simulation results shown in Figure 9. As we know, the role of early reflections in acoustical perception (speech intelligibility, clarity, spaciousness) is very significant, such that accurate prediction of these reflections is quite important.
CONCLUSIONS AND FUTURE RESEARCH

In this paper a new method for wave based room acoustical simulations is used to simulate three simple geometrical configurations with both locally and non-locally reacting boundary conditions. Although for the non-locally reacting character of boundaries a rather extreme case was chosen, the results indicate the importance of taking into account the angle-dependent reflection characteristics of boundaries. As can be concluded from the shown simulations a significant difference is found in the early reflections, which are important for the acoustical perception in a room. The differences are only qualitatively shown and therefore further research has to be conducted to give a quantitatively comparison. Also, the perceptual significance of the results has to be investigated experimentally.

REFERENCES